



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 11

Valuation of Bonds with Embedded Options

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VALUATION OF BONDS WITH EMBEDDED OPTIONS



TYPES OF EMBEDDED OPTIONS

- The embedded options can be
- A call option
- A put option
- Extendible bonds
- Estate put
- Provisions for a sinking fund.



SIMPLE CALLABLE BONDS

- **Callable bonds** give the *issuer of the bond*, the option to call back the bond on terms contained in the offer document.
- Call option will be invoked by the issuer in the regime of falling interest rates.
- Most callable bonds have a lockout period during which the bond cannot be called.

PUTTABLE BONDS

- **Puttable bonds allow the investor to put (sell) the bond back to the issuer prior to maturity.**



EXTENDIBLE BONDS

- A related bond is an extendible bond, which allows the investor to extend the maturity of the bond.
- An extendible bond can be evaluated as a puttable bond with longer maturity (i.e., the maturity if the bond is extended).
- A two-year, 3% bond extendible for an additional year at the same coupon rate would be valued the same as an otherwise identical three-year puttable (European style) bond with a lockout period of two years.

COMPLEX OPTIONS: ESTATE PUT

- An estate put includes a provision that allows the heirs of an investor to put the bond back to the issuer upon the death of the investor.
- The value of this contingent put option is inversely related to the investor's life expectancy; the shorter the life expectancy, the higher the value.

SINKING FUND BONDS

- **Sinking fund bonds (sinkers) require the issuer to set aside funds periodically to retire the bond (a sinking fund). This provision reduces the credit risk of the bond.**
- **Sinkers typically have several related issuer options (e.g., call provisions, acceleration provisions).**

RELATIONSHIPS BETWEEN THE VALUES OF A CALLABLE OR PUTTABLE BOND, THE UNDERLYING OPTION-FREE (STRAIGHT) BOND, AND THE EMBEDDED OPTION.



CASE OF CALLABLE BOND

- In essence, the holder of a callable bond owns an option-free (straight) bond and is also short a call option written on the bond.
- The value of the callable bond ($V_{callable}$) is, therefore, simply the difference between the value of a straight bond ($V_{straight}$) and the value of the embedded call option (V_{call}):
- $$(V_{callable}) = (V_{straight}) - (V_{call})$$

CASE OF PUTTABLE BOND

- Conversely, investors are willing to pay a premium for a puttable bond, since its holder effectively owns an option-free bond plus a put option.
- The value of a puttable bond can be expressed as:
- $V_{puttable} = V_{straight} + V_{put}$
- Rearranging, the value of the embedded put option can be stated as:
- $V_{put} = V_{puttable} - V_{straight}$

VALUING BONDS WITH EMBEDDED OPTIONS WITH BINOMIAL INTEREST RATE TREE



CASE OF CALLABLE BOND

- The basic process for valuing a callable (or puttable) bond is similar to the process for valuing a straight bond.
- We compute the value of the bond at different points in time at each node of the tree and determine whether the embedded option is in-the-money.
- If it is in the money, it is assumed to be exercised and cash flows adjusted accordingly.

CALL RULE

- Thus, at any node, if the call is exercisable due to the computed price exceeding the strike price, the computed price at that node is replaced by the strike price.
- Hence, when valuing a callable bond, the value at any node where the bond is callable must be either the price at which the issuer will call the bond (i.e., the call price) or the computed value if the bond is not called, whichever is lower. This is known as the call rule.

PUT RULE

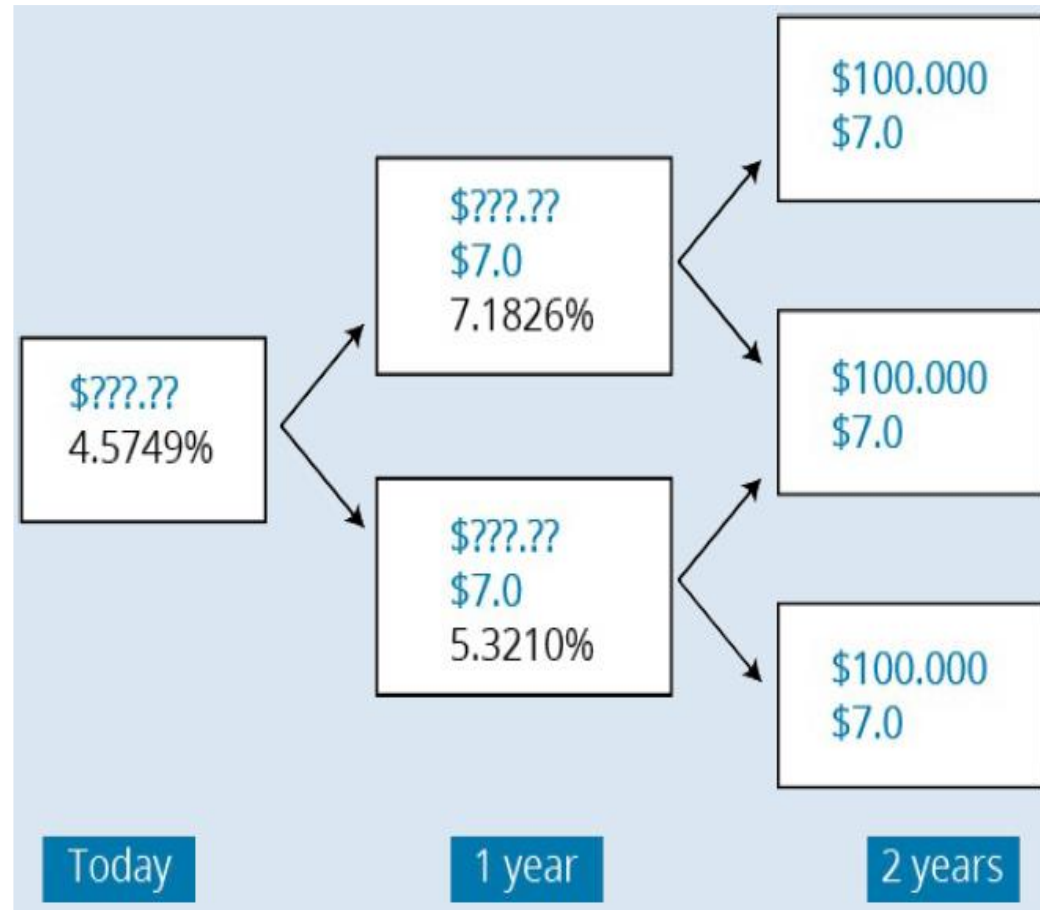
- Similarly, for a puttable bond, the value used at any node corresponding to a put date must be either the price at which the investor will put the bond (i.e., the put price) or the computed value if the bond is not put, **whichever is higher**. This is known as the put rule.

EXAMPLE

Consider a two-year, 7% annual-pay, \$100 par bond callable in one year at \$100.

Also consider a two-year, 7% annual-pay, \$100 par bond puttable in one year at \$100.

Interest rate tree at 15% assumed volatility is as given. Value the bonds.



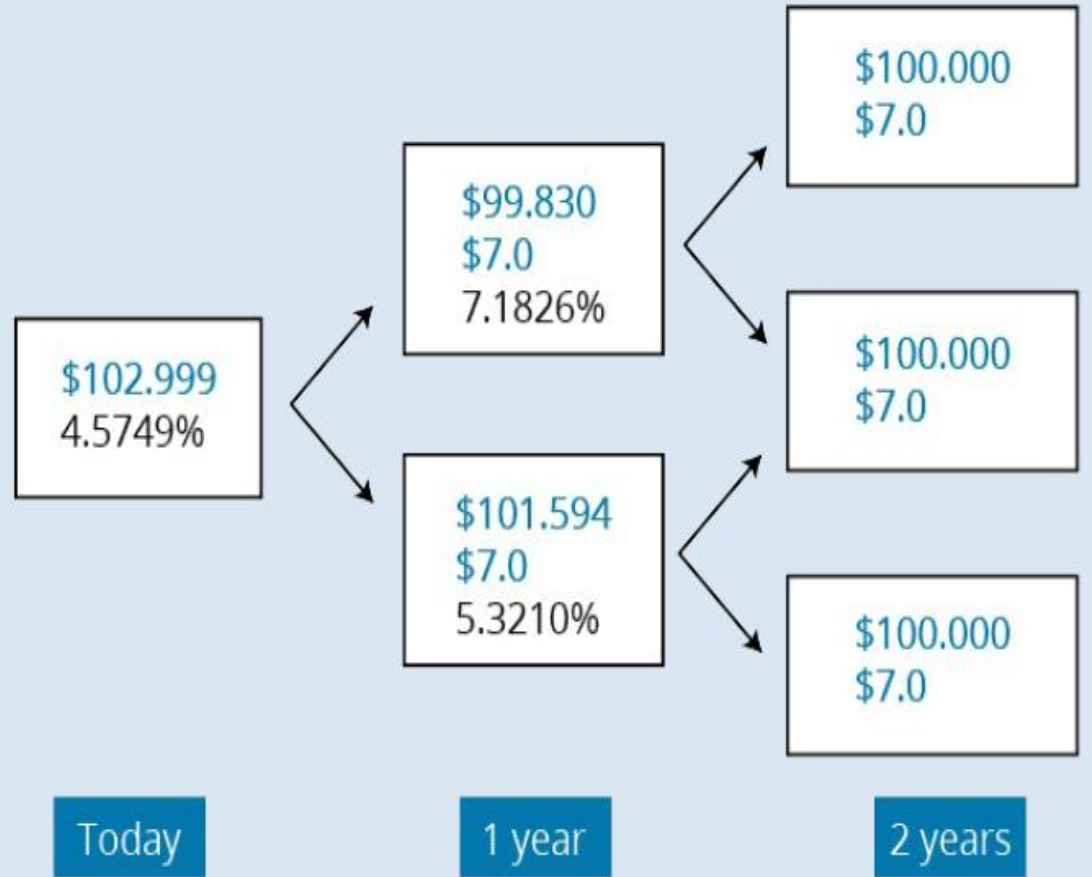
STRAIGHT BOND

$$V_{1,U} = \frac{\$100 + \$7}{1.071826} = \$99.830$$

$$V_{1,L} = \frac{\$100 + \$7}{1.053210} = \$101.594$$

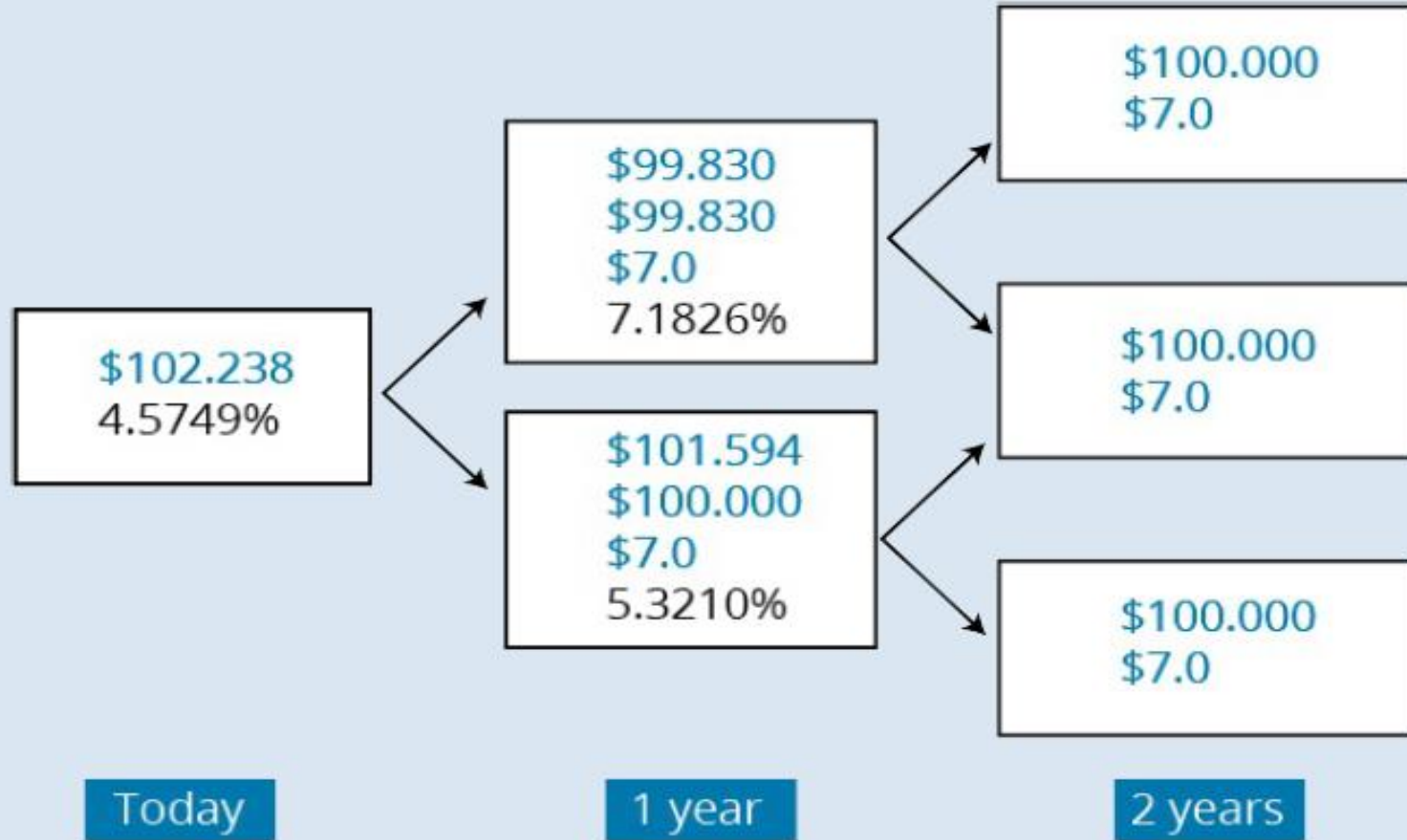
$$V_0 = \frac{1}{2}$$

$$\times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$100 + \$7}{1.045749} \right] = \$102.999$$



- $V_{1,U} = \frac{\$100 + \$7}{1.071826} = \$99.830$
- $V_{1,L} = \frac{\$100 + \$7}{1.053210} = \$101.594$
- $V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$100 + \$7}{1.045749} \right]$
- $= \$102.999$

VALUE OF THE CALLABLE BOND



- $V_{1,U} = \frac{107}{1.071826} = \$99.830$
- $V_{1,L} = \$100$ instead of $\$101.594$
- $V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$100.00 + \$7}{1.045749} \right]$
- $= \$102.238$

- The call rule (call the bond if the price exceeds \$100) is reflected in the boxes in the completed binomial tree, **where the second line of the boxes at the one-year node is the lower of the call price or the computed value.**
- **For example, the value of the bond in one year at the lower node is \$101.594. However, in this case, the bond will be called, and the investor will only receive \$100. Therefore, for valuation purposes, the value of the bond in one year at this node is \$100.**

- Similarly, for a puttable bond, the put rule is to put the bond if the value falls below \$100. The put option would therefore be exercised at the upper-node in year 1 and hence the \$99.830 computed value is replaced by the exercise price of \$100.
- $V_{1,U} = \$100$
- $V_{1,L} = (107/1.053210) = \$101.594$
- $V_0 = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.045749} + \frac{\$101.594 + \$7}{1.045749} \right]$
- $= \$103.081$

- Value of the embedded options:
- $V_{call} = V_{straight} - V_{callable}$
- $= \$102.999 - \$102.238 = \$0.76$
- $V_{put} = V_{puttable} - V_{straight}$
- $= \$103.081 - \$102.999 = 0.082$

EFFECT OF INTEREST RATE VOLATILITY ON THE VALUE OF A CALLABLE OR PUTTABLE BONDS



STRAIGHT BONDS & VOLATILITY

- Although, the value of a straight bond is affected by changes in the level of interest rates,
- the value of the straight bond is unaffected by changes in the volatility of interest rates.



OPTION VALUES & VOLATILITY

- **Option values are positively related to the volatility of their underlying.**
- **Accordingly, when interest rate volatility increases, the values of both call and put options increase.**

- Thus, when interest rate volatility increases, both V_{call} , V_{put} increase,
- Since $V_{callable} = V_{straight} - V_{call}$
- $V_{puttable} = V_{straight} + V_{put}$
- the value of a callable bond (where the investor is short the call option) decreases and
- the value of a puttable bond (where the investor is long the put option) increases.

EFFECT OF CHANGES IN THE LEVEL OF THE YIELD CURVE ON THE VALUE OF A CALLABLE OR PUTTABLE BOND



EFFECT OF LEVEL OF INTT RATES: CALLABLE BOND

- There are several effects of a change in interest rate on a callable bond:
- A. As interest rates decline, the market price of the straight bond rises since the required return by investors across bonds falls.

- **B. A rise in the price of the bonds raises the probability of exercise of the call by the issuer in the callable bond.**
- **Hence, the value of the call in the callable bond rises.**
- **Because the investor is short in the call, the price of the callable bond decreases on this count.**

- **C. If the bond's market price rises above the exercise price (due to the fall in market interest rates), then the issuer may call back the bond and the investor has to deliver the bond back to the issuer at the exercise price (which is lower than market price).**
- **Thus, the call option in the callable bond limits the bond's upside, so the value of a callable bond rises less rapidly than the value of an otherwise-equivalent straight bond.**

EFFECT OF LEVEL OF INTT RATES: PUTTABLE BOND

- As interest rates increase, the long put in a puttable bond hedges against the loss in value.
- Thus, the value of a puttable bond falls less rapidly than the value of an otherwise-equivalent straight bond.

SUMMARY: CALLABLE BOND

- Thus, if interest rates decline:
- The value of the straight bond increases.
- The value of the call option in the callable bond increases;
- Thus, both, the value of the call & the value of the straight bond are inversely related to the level of interest rates
- The callability of the bond also limits the bond's upside.
- On all these counts, the value of the callable bond increases slower than that of a straight bond.
- **The value of the callable bond changes inversely but at a slower rate than the straight bond.**

SUMMARY: PUTTABLE BONDS

- If interest rates increase:
- The value of the straight bond decreases.
- The probability of exercise of the put increases.
- The value of the put option in the puttable bond increases.
- Thus, the value of the put changes directly & the value of the straight bond inversely with the change in level of interest rates.
- The value of the puttable bond decreases slower than that of a straight bond because the holder of a puttable bond is long in the put.